INTRODUCTION

Because most students that use Understanding Healthcare Financial Management will be conducting time value analyses on spreadsheets, most of the text discussion focuses on spreadsheet solutions. Still, some students will be using calculators to solve time value problems. This tutorial, which focuses on calculator solutions, was prepared to assist those students.

FUTURE VALUE OF A LUMP SUM (COMPOUNDING)

The process of going from today's values, or present values, to future values is called compounding, and lump sum compounding deals with a single starting cash flow. Suppose that the manager of Meridian Clinic deposits $100 in a bank account that pays 5 percent interest per year. How much would be in the account at the end of five years?

Regular calculator solution:

A regular (nonfinancial) calculator can be used, either by multiplying the PV by \((1 + I)\) for \(N\) times or by using the exponential function to raise \((1 + I)\) to the \(N\)th power and then multiplying the result by the PV. The easiest way to find the future value of $100 after five years when compounded at 5 percent is to enter $100, then multiply this amount by 1.05 five times. If the calculator is set to display two decimal places, the answer would be $127.63:

\[
0 \quad 5\% \quad 1 \quad 2 \quad 3 \quad 4 \quad 5
\]

\[
\begin{align*}
\$100 \times 1.05 & \\
\times 1.05 & \\
\times 1.05 & \\
\times 1.05 & \\
\times 1.05 & = \$127.63
\end{align*}
\]

As denoted by the arrows, compounding involves moving to the right along the time line.
Financial calculator solution:

Financial calculators are preprogrammed to solve many types of time value problems. In effect, the future value calculation is programmed directly into the memory, so the user merely has to input the requisite starting values. Using a financial calculator, the future value is found using these time value input keys:

\[
\begin{array}{cccccc}
N & I & PV & PMT & FV \\
\end{array}
\]

which correspond to the five time value variables:

- \(N\) = number of periods.
- \(I\) = interest rate per period.
- \(PV\) = present value.
- \(PMT\) = payment (used only when the problem involves a series of equal cash flows).
- \(FV\) = future value.

On some financial calculators, the keys are buttons on the face of the calculator; on others, the time value variables are shown on the display after accessing the time value menu. Also, some calculators use different symbols to represent the number of periods and interest rate. For example, both lower and upper cases are used for \(N\) and \(I\), while other calculators use \(N/YR\) and \(I/YR\) or \(I%/YR\) or some other variation.

Financial calculators today are quite powerful in that they can easily solve relatively complex time value of money problems, such as when intraperiod cash flows occur. To focus on concepts rather than mechanics, all the illustrations in this tutorial assume that cash flows occur at the end or beginning of a period, and that there is only one cash flow per period. Thus, to follow the illustrations, **financial calculators must be set to one period per year**, and it is not necessary to use the calendar function.

Note that this problem deals with only four of the time value variables. Three of the variables will be known, and the calculator will solve for the fourth, unknown variable. When bond valuation is discussed later in the tutorial, all five variables will be included in the analysis.
To find the future value of $100 after five years when invested at 5 percent interest using a financial calculator, just enter PV = -100, I = 5, and N = 5, then press the FV key. The answer, 127.63 (rounded to two decimal places), will appear:

<table>
<thead>
<tr>
<th>Inputs</th>
<th>5</th>
<th>5</th>
<th>-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>I</td>
<td>PV</td>
<td>PMT</td>
</tr>
<tr>
<td>Output</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that most financial calculators require that cash flows be designated as either inflows or outflows (entered as either positive or negative values). Applying this logic to the illustration, Meridian deposits the initial amount, which is an outflow to the business, and takes out, or receives, the ending amount, which is an inflow to the business. (If the PV was entered as 100, a positive value, the answer on a calculator using sign convention would be displayed as -127.63.)

Note that some calculators require the user to press a Compute key before pressing the FV key. Also, financial calculators permit specifying the number of decimal places that are displayed, even though 12, or more, significant digits are actually used in the calculations. Two places are generally used for answers in dollars or percentages, and four places for decimal answers. The final answer, however, should be rounded to reflect the accuracy of the input values; it makes no sense to say that the return on a particular investment is 14.63827 percent when the cash flows are highly uncertain. The nature of the analysis dictates how many decimal places should be displayed.

**PRESENT VALUE OF A LUMP SUM (DISCOUNTING)**

Suppose that GroupWest Health Plans, which has premium income reserves to invest, has the opportunity to purchase a low-risk security that will pay $127.63 at the end of five years. A local bank is currently offering 5 percent interest on a five-year certificate of deposit (CD), and GroupWest's managers regard the security being offered as being as safe as the bank CD. The 5 percent interest rate available on the bank CD is GroupWest's opportunity cost rate. How much would GroupWest be willing to pay for this security?
In the previous section, we learned that an initial amount of $100 invested at 5 percent per year would be worth $127.63 at the end of five years. Thus, GroupWest should be indifferent to the choice between $100 today and $127.63 to be received after five years. Today's $100 is defined as the present value, or \( PV \), of $127.63 due in five years when the opportunity cost rate is 5 percent. Finding present values is called discounting, and it is simply the reverse of compounding: If the PV is known, compound to find the FV; if the FV is known, discount to find the PV. Here are the solution techniques used to solve this discounting problem.

**Regular calculator solution:**

Enter $127.63 and divide it five times by 1.05:

\[
\begin{array}{cccccc}
\hline
0 & 5\% & 1 & 2 & 3 & 4 & 5 \\
\hline
$100 & \div & 1.05 & \div & 1.05 & \div & 1.05 & \div & 1.05 & \div & 1.05 \div 127.63 \\
\hline
\end{array}
\]

As shown by the arrows, discounting is moving to the left along a time line.

**Financial calculator solution:**

```
Inputs                5               5                                                        127.63
N          I          PV         PMT         FV
Output                                           = -100.00
```

**SOLVING FOR INTEREST RATE AND TIME**

In our examples thus far, four time value analysis variables have been used: PV, FV, I, and N. Specifically, the interest rate, I, and the number of years, N, plus either PV or FV have been initially given. However, if the values of any three of the variables are known, the value of the fourth can be found.
Solving for Interest Rate (I)

Suppose that Family Practice Associates (FPA), a primary care physicians’ group practice, can buy a bank CD for $78.35 that will return $100 after five years. In this case PV, FV, and N are known, but I, the interest rate that the bank is paying, is not known. Such problems are solved in this way:

Time line:

\[
\begin{array}{cccccc}
0 & ? & 1 & 2 & 3 & 4 & 5 \\
\downarrow & & & & & & \\
-78.35 & & & & & & 100 \\
\end{array}
\]

Financial calculator solution:

Inputs  
\begin{array}{cccc}
5 & I & -78.35 & 100 \\
\end{array}

Output  
\begin{array}{cccc}
N & I & PV & PMT & FV \\
5.0 & \\
\end{array}

Solving for Time (N)

Suppose that the bank told FPA that a CD pays 5 percent interest each year, that it costs $78.35, and that at maturity the group would receive $100. How long must the funds be invested in the CD? In this case, PV, FV, and I are known, but N, the number of periods, is not known.

Time line:

\[
\begin{array}{cccccccc}
0 & 5\% & 1 & 2 & \ldots & N-1 & N \\
\downarrow & & & & & & & \\
-78.35 & & & & & & $100 \\
\end{array}
\]

Financial calculator solution:

Inputs  
\begin{array}{cccc}
5 & I & -78.35 & 100 \\
\end{array}

Output  
\begin{array}{cccc}
N & I & PV & PMT & FV \\
5.0 & \\
\end{array}
ANNUITIES

Whereas lump sums are single cash flows, an annuity is a series of equal cash flows at fixed intervals for a specified number of periods. Annuity cash flows, which often are called payments and given the symbol PMT, can occur at the beginning or end of each period. If the payments occur at the end of each period, the annuity is an ordinary, or deferred, or regular annuity. If payments are made at the beginning of each period, the annuity is an annuity due.

Ordinary Annuities

A series of equal payments at the end of each period constitute an ordinary annuity. If Meridian Clinic were to deposit $100 at the end of each year for three years in an account that paid 5 percent interest per year, how much would Meridian accumulate at the end of three years? The answer to this question is the future value of the annuity.

Regular calculator solution:

One approach is to treat each individual cash flow as a lump sum, compound it to Year 3, then sum the future values:

<table>
<thead>
<tr>
<th>0</th>
<th>5%</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$100</td>
<td>$100</td>
<td>$100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>105</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>110.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$315.25</td>
</tr>
</tbody>
</table>

Financial calculator solution:

\[
\begin{array}{cccccc}
\text{Inputs} & 3 & 5 & \text{PV} & \text{PMT} & \text{FV} \\
\text{Output} & & & & \text{315.25} \\
\end{array}
\]

In annuity problems, the PMT key is used in conjunction with either the PV or FV key.
Suppose that Meridian Clinic was offered the following alternatives: (a) a three-year annuity with payments of $100 at the end of each year, or (b) a lump sum payment today. Meridian has no need for the money during the next three years. If it accepts the annuity, it would deposit the payments in an account that pays 5 percent interest per year. Similarly, the lump sum payment would be deposited into the same account. How large must the lump sum payment be today to make it equivalent to the annuity? In other words, what is the present value of the annuity?

Regular calculator solution:

Financial calculator solution:

Annuities Due

If the three $100 payments in the previous example had been made at the beginning of each year, the annuity would have been an annuity due. When compared to an ordinary annuity, each payment is shifted to the left one year. Because the payments come in faster, an annuity due is more valuable than an ordinary annuity.

Regular calculator solution:
In the case of an annuity due, as compared with an ordinary annuity, all the cash flows are compounded for
one additional period, and hence its future value is greater than the future value of a similar ordinary annuity
by \((1 + i)\). Thus, the future value of an annuity due also can be found as follows:

\[
FV\text{ (Annuity due)} = FV\text{ of a regular annuity} \times (1 + i)
\]

\[
= $315.25 \times 1.05 = $331.01.
\]

Financial calculator solution:

Most financial calculators have a switch or key marked DUE or BEGIN that permits the switching of the mode
from end-of-period payments (ordinary annuity) to beginning-of-period payments (annuity due). When the
beginning-of-period mode is activated, the display will normally indicate the changed mode with the word
BEGIN or another symbol. To deal with annuities due, change the mode to beginning of period and proceed
as before. Because most problems will deal with end-of-period cash flows, do not forget to switch the
calculator back to the END mode.

The present value of an annuity due is found in a similar manner.

Regular calculator solution:
The present value of an annuity due can be thought of as the present value of an ordinary annuity that is compounded for one period, so it also can be found as follows:

\[
PV \text{ (Annuity due)} = PV \text{ of a regular annuity} \times (1 + I)
\]

\[
= $272.32 \times 1.05 = $285.94.
\]

Financial calculator solution:

Activate the beginning of period mode (i.e., the BEGIN mode), then proceed as before. Again, because most problems will deal with end-of-period cash flows, do not forget to switch back to the END mode.

UNEVEN CASH FLOW STREAMS

The definition of an annuity includes the words "constant amount," so annuities involve cash flows that are the same in every period. Although some financial decisions, such as bond valuation, do involve constant cash flows, most important healthcare financial analyses involve uneven, or nonconstant, cash flows. For example, the financial evaluation of a proposed outpatient clinic or MRI facility rarely involves constant cash flows.

In general, the term payment \((PMT)\) is reserved for annuity situations, in which the cash flows are constant, and the term cash flow \((CF)\) denotes either lump sums or uneven cash flows. Financial calculators are set up to follow this convention. When dealing with uneven cash flows, CF functions, rather than the PMT key, are used.

Present Value

The present value of an uneven cash flow stream is found as the sum of the present values of the individual cash flows of the stream. For example, suppose that Wilson Memorial Hospital is considering the purchase of a new x-ray machine. The hospital’s managers forecast that the operation of the new machine would produce the following stream of cash inflows (in thousands of dollars):

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 & 5 \\
100 & 120 & 150 & 180 & 250 \\
\end{array}
\]
What is the present value of the new x-ray machine investment if the appropriate discount rate (i.e., the opportunity cost rate) is 10 percent?

*Regular calculator solution:*

The PV of each individual cash flow can be found using a regular calculator, then these values are summed to find the present value of the stream, $580,950:

<table>
<thead>
<tr>
<th>Time</th>
<th>Cash Flow</th>
<th>PV Factor (10%)</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>120</td>
<td>0.9091</td>
<td>109.10</td>
</tr>
<tr>
<td>2</td>
<td>150</td>
<td>0.8264</td>
<td>123.96</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
<td>0.7513</td>
<td>135.23</td>
</tr>
<tr>
<td>4</td>
<td>250</td>
<td>0.6830</td>
<td>170.75</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0.6209</td>
<td>0</td>
</tr>
</tbody>
</table>

$\text{PV} = 100 + 109.10 + 123.96 + 135.23 + 170.75 + 0 = 580.95$

*Financial calculator solution:*

The present value of an uneven cash flow stream can be solved with most financial calculators by using the following steps:

- Input the individual cash flows, in chronological order, into the cash flow registers, usually designated as $\text{CF}_0$ and $\text{CF}_j$ ($\text{CF}_1$, $\text{CF}_2$, $\text{CF}_3$, and so on) or just $\text{CF}_j$ ($\text{CF}_0$, $\text{CF}_1$, $\text{CF}_2$, $\text{CF}_3$, and so on).
- Enter the discount rate.
- Push the NPV key.

For this problem, enter 0, 100, 120, 150, 180, and 250 in that order into the calculator's cash flow registers; enter $I = 10$; then push NPV to obtain the answer, 580.95. Note that an implied cash flow of zero is entered for $\text{CF}_0$.

Note that when dealing with the cash flow registers, the term NPV, rather than PV, is used to represent present value. The letter N in NPV stands for the word net, so NPV is the abbreviation for net present value.
present value. Net present value means the sum or net of the present values of a cash flow stream. In general, the stream will consist of both inflows and outflows, but the stream here contains all inflows.

Also, annuity cash flows within any uneven cash flow stream can be entered into the cash flow registers most efficiently on most calculators by using the Nj key. This key allows the user to specify the number of times a constant payment occurs within the stream. (Some calculators prompt the user to enter the number of times each cash flow occurs.)

Finally, amounts entered into the cash flow registers remain in those registers until they are cleared. Thus, if a problem had been previously worked with eight cash flows, and a problem is worked with only four cash flows, the calculator assumes that the final four cash flows from the first calculation belonged to the second calculation. Be sure to clear the cash flow registers before starting a new problem.

**Future Value**

The future value of an uneven cash flow stream is found by compounding each payment to the end of the stream then summing the future values.

*Regular calculator solution:*

The future value of each individual cash flow can be found, using a regular calculator, by summing these values to find the future value of the stream, $935,630:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$100</td>
<td>$120</td>
<td>$150</td>
<td>$180</td>
<td>$250</td>
<td>$250</td>
</tr>
<tr>
<td>10%</td>
<td>198.00</td>
<td>181.50</td>
<td>159.72</td>
<td>146.41</td>
<td>$935.63</td>
<td></td>
</tr>
</tbody>
</table>

*Financial calculator solution:*

Some financial calculators have a net future value key (NFV) that, after the cash flows have been entered into
the cash flow registers, can be used to obtain the future value of an uneven cash flow stream. However, there is generally more interest in the present value of a cash flow stream than in its future value because the present value represents the value of the investment today, which then can be compared to the cost of the investment, be it a stock, bond, x-ray machine, or new clinic.

**USING TIME VALUE ANALYSIS TO MEASURE FINANCIAL RETURNS**

In most investments, an individual or business spends cash today with the expectation of receiving cash in the future. The financial attractiveness of such investments is measured by *financial return*, or just *return*. There are two basic ways of expressing financial return: (1) in dollar terms and (2) in percentage terms.

To illustrate the concept, let’s reexamine the cash flows expected to be received if Wilson Memorial Hospital buys its new x-ray machine (shown on the time line in thousands of dollars). In the last section, we determined that the present value of these flows, when discounted at a 10 percent rate, is $580,950:

```
0      10%      1      2      3      4      5
├───────┼───────┼───────┼───────┼───────┤
│$100   │$120   │$150   │$180   │$250   │
└──────────────────────────────────┘

$ 90.91 <
99.17 <
112.70 <
122.94 <
155.23 <
$580.95
```

**Dollar Return**

The $580,950 calculated above represents the present value (in financial terms) of the cash flows that the x-ray machine is *expected* to provide to Wilson Memorial Hospital. Note that these cash flows are not known with certainty but rather represent the best estimates of the hospital’s managers.

To measure the *dollar return* on the investment, the cost of the x-ray machine must be compared to the present value of the expected benefits (the cash inflows). If the machine is expected to cost $500,000, and the present value of the inflows is $580,950, then the expected dollar return on the machine is $580,950.
$\$500,000 = $80,950$. Note that this measure of dollar return incorporates time value through the discounting process. Also, the opportunity cost inherent in the use of the $500,000 is accounted for because the 10 percent discount rate reflects the return that could be earned on alternative investments of similar risk. Thus, the x-ray machine is expected to produce a $80,950 return above that required for its riskiness as accounted for by the 10 percent opportunity cost rate.

The dollar return process can be combined into a single calculation by adding the cost of the x-ray machine to the time line:

Financial calculator solution:

Now, with the investment outlay (cost) added to the time line, the following cash flows would be entered in the cash flow registers: -500, 100, 120, 150, 180, and 250, in that order. Then, enter $I = 10$ and push NPV to obtain the answer, 80.95.

Rate of Return

The second way to measure the financial return on an investment is by rate of return, or percentage return. This measures the interest rate that must be earned on the investment outlay to generate the expected cash inflows. In other words, this measure provides the expected periodic rate of return on the investment. If the cash flows are annual, as in this example, the rate of return is an annual rate. In effect, we are solving for $I$—the interest rate that equates the present value of the cash inflows to the dollar amount of the cash outlay.

Mathematically, if the present value of the cash inflows equals the investment outlay, then the NPV of
the investment is forced to $0. This relationship is shown here:

<table>
<thead>
<tr>
<th></th>
<th>IRR%</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$500.00</td>
<td>$100</td>
<td>$120</td>
<td>$150</td>
<td>$180</td>
<td>$250</td>
<td></td>
</tr>
</tbody>
</table>

Note that the rate of return on an investment, particularly an investment in plant or equipment, typically is called the *internal rate of return* (IRR). Although a trial-and-error procedure could be used on a regular calculator to determine the rate of return, it is better to use a financial calculator or spreadsheet.

*Financial calculator solution:*

Use the same cash flows that were entered to solve for NPV: -500, 100, 120, 150, 180, and 250. However, now push the IRR button to obtain the answer, 15.3 percent.

**SEMIANNUAL AND OTHER COMPOUNDING PERIODS**

In all the examples thus far, the assumption was that interest is compounded once a year, or annually. This is called *annual compounding*. Suppose, however, that Meridian Clinic puts $100 into a bank account that pays 6 percent annual interest but is compounded *semiannually*. How much would the clinic accumulate at the end of one year, two years, or some other period? Semiannual compounding means that interest is paid each six months, so interest is earned more often than under annual compounding.

To illustrate semiannual compounding, assume that the $100 is placed into the account for three years. The following situation occurs under *annual* compounding:

*Regular calculator solution:*
$100 \times 1.06 \times 1.06 \times 1.06 = 119.10$

Financial calculator solution:

Inputs: 3 6 -100

Output: 119.10

Now, consider what happens under semiannual compounding. Because interest rates usually are stated as annual rates, this situation would be described as 6 percent interest, compounded semiannually.

With semiannual compounding, $N = 2 \times 3 = 6$ semiannual periods, and $I = 6 / 2 = 3\%$ per semiannual period.

Regular calculator solution:

$100 \times 1.03 \times 1.03 \times 1.03 \times 1.03 \times 1.03 \times 1.03 = 119.41$

Financial calculator solution:

Inputs: 6 3 -100

Output: 119.41

The $100$ deposit grows to $119.41$ under semiannual compounding, but only to $119.10$ under annual compounding. This result occurs because interest on interest is being earned more frequently under semiannual compounding.

Throughout the economy, different compounding periods are used for different types of investments. For example, bank accounts often compound interest monthly or daily, most bonds pay interest semiannually, and stocks generally pay quarterly dividends. Furthermore, the cash flows stemming from capital investments, such as new hospital wings or diagnostic equipment, can be analyzed in monthly, quarterly, or annual periods.
or even in some other interval. Time value analyses with different compounding periods must be put on a common basis for comparison, which is accomplished by the effective annual rate.

To begin the comparison, note that the stated interest rate in the Meridian Clinic semiannual compounding example is 6 percent, while the effective annual rate is the rate that produces the same ending value under annual compounding. In the example, the effective annual rate is the rate that would produce a future value of $119.41 at the end of Year 3 under annual compounding. The solution is 6.09 percent:

\[
\begin{array}{cccccc}
\text{Inputs} & N & I & PV & PMT & FV \\
\text{Output} & & & & & = 6.09
\end{array}
\]

Thus, if one bank offered to pay 6 percent interest with semiannual compounding on a savings account, while another offered 6.09 percent with annual compounding, both banks would be paying the same effective annual rate because the ending value is the same under both sets of terms:

\[
\begin{array}{cccccc}
\text{Semiannual periods} & 0 & 3\% & 1 & 2 & 3 & 4 & 5 & 6 \\
$100 \times 1.03 & \times 1.03 & \times 1.03 & \times 1.03 & \times 1.03 & \times 1.03 = $119.41
\end{array}
\]

\[
\begin{array}{cccccc}
\text{Years} & 0 & 6.09\% & 1 & 2 & 3 \\
$100 \times 1.0609 & \times 1.0609 & \times 1.0609 = $119.41
\end{array}
\]

In general, the effective annual rate (EAR) can be determined, given the stated rate and number of compounding periods per year, by using this equation:

\[
\text{Effective annual rate (EAR)} = (1 + \frac{I_{\text{stated}}}{M})^M - 1.0.
\]

Here, \(I_{\text{stated}}\) is the stated (i.e., the annual) interest rate and \(M\) is the number of compounding periods per year.

The term \(\frac{I_{\text{stated}}}{M}\) is the periodic interest rate, so the EAR equation can be restated as:
Effective annual rate (EAR) = \((1 + \text{Periodic rate})^M - 1.0\).

To illustrate use of the EAR equation, consider that the effective annual rate when the stated rate is 6 percent and semiannual compounding occurs is 6.09 percent:

\[
\text{EAR} = (1 + 0.06/2)^2 - 1.0
\]
\[
= (1.03)^2 - 1.0
\]
\[
= 1.0609 - 1.0 = 0.0609 = 6.09\%.
\]

Most financial calculators are programmed to calculate EAR, or given an EAR, to find the stated rate. This process is called interest rate conversion. In general, enter the stated rate first, then the number of compounding periods per year, and finally the effective percent.

Compounding periods (other than annual) also can occur when dealing with annuities. To illustrate the concept, first consider the case of an ordinary annuity of $100 per year for three years discounted at 8 percent, compounded annually.

*Time line:*

\[
\begin{array}{cccccc}
0 & 8\% & 1 & 2 & 3 \\
? & $100 & $100 & $100 \\
\end{array}
\]

*Financial calculator solution:*

Inputs 3 8 100

\[
\begin{array}{cccc}
\text{N} & \text{I} & \text{PV} & \text{PMT} & \text{FV} \\
\hline
\end{array}
\]

Output = -257.71

Suppose that the annuity calls for payments of $50 every six months for three years, and the interest rate is 8 percent, compounded semiannually:

*Time line:*

\[
\begin{array}{cccccccc}
0 & 8\% & 1 & 2 & 3 & 4 & 5 & 6 \\
? & $50 & $50 & $50 & $50 & $50 & $50 & $50 \\
\end{array}
\]
Financial calculator solution:

<table>
<thead>
<tr>
<th>Inputs</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>6</td>
<td>I</td>
<td>4</td>
<td>PV</td>
</tr>
<tr>
<td>PMT</td>
<td>50</td>
<td>FV</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Output = -262.11

Semiannual payments come in earlier than annual payments, so the $50 semiannual annuity is a little more valuable than the $100 annual annuity. However, an annuity with annual payments, but with semiannual compounding, cannot be treated in the same way because the discount rate period must match the annuity period. Thus, annual payments must be discounted using an annual rate, and when semiannual compounding occurs with annual payments, the EAR (instead of the stated rate) must be applied.

AMORTIZED LOANS

One important application of time value analysis involves loans that are to be paid off in equal installments over time, such as automobile loans, home mortgage loans, and most business debt other than very short-term loans and long-term bonds. If a loan is to be repaid in equal periodic amounts—monthly, quarterly, or annually—it is said to be an amortized loan. The word amortize comes from the Latin mors, meaning death, so an amortized loan is one that is killed off over time.

To illustrate the concept, suppose Santa Fe Healthcare System borrows $1 million from the Bank of New Mexico that will be repaid in three equal installments at the end of each of the next three years. The bank is to receive 6 percent interest on the loan balance that is outstanding at the beginning of each year. The first task in analyzing the loan is to determine the amount Santa Fe must repay each year, or the annual payment. To find this amount, recognize that the loan represents the present value of an annuity of PMT dollars per year for three years, discounted at 6 percent.
Financial calculator solution:

Inputs              3               6           1000000
                     N          I          PV         PMT         FV
Output                                                               = -374,110

Therefore, if Santa Fe pays the bank $374,110 at the end of each of the next three years, the percentage cost to the borrower, and the rate of return to the lender, will be 6 percent.

Each payment consists partly of interest and partly of repayment of principal. This breakdown is given in the amortization schedule shown in Table 1. The interest component is largest in the first year, and it declines as the outstanding balance of the loan is reduced over time. For tax purposes, a taxable business borrower reports the interest payments in Column 3 as a deductible cost each year, while the lender reports these same amounts as taxable income.

Table 1
Loan Amortization Schedule

<table>
<thead>
<tr>
<th>Year</th>
<th>Beginning Amount (1)</th>
<th>Payment (2)</th>
<th>Interesta (3)</th>
<th>Repayment of Principalb (4)</th>
<th>Remaining Balance (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1,000,000</td>
<td>$374,110</td>
<td>$60,000</td>
<td>$314,110</td>
<td>$685,890</td>
</tr>
<tr>
<td>2</td>
<td>685,890</td>
<td>374,110</td>
<td>41,153</td>
<td>332,957</td>
<td>352,933</td>
</tr>
<tr>
<td>3</td>
<td>352,933</td>
<td>374,110</td>
<td>21,177</td>
<td>352,933</td>
<td>0</td>
</tr>
</tbody>
</table>

| Sum   | $1,122,330           | $122,330    | $1,000,000    |

aInterest is calculated by multiplying the loan balance at the beginning of each year by the interest rate. Therefore, interest in Year 1 is $1,000,000 x 0.06 = $60,000; in Year 2 is $685,890 x 0.06 = $41,153; and in Year 3 is $352,933 x 0.06 = $21,177.

bRepayment of principal is equal to the payment of $374,110 minus the interest charge for each year.

Financial calculators are often programmed to calculate amortization schedules; simply key in the inputs, then press one button to get each entry in Table 1.

FRACTIONAL TIME PERIODS
In all of the examples in this tutorial, we have assumed that payments occur at the beginning or the end of periods, but not within a period. However, many situations arise in which cash flows do occur within periods. For example, Meridian Clinic might deposit $100 in a bank that pays 10 percent interest compounded annually, and leave it in the bank for nine months, or 0.75 years. How much would be in the account at the end?

*Time line:*

\[
\begin{array}{c|c|c|c}
0 & 10\% & 1 \\
\hline
-100 & ? & 107.41 \\
\end{array}
\]

*Regular calculator solution:*

\[
FV_n = 100 \times (1.10)^{0.75} = 100 \times (1.0741) = 107.41.
\]

*Financial calculator solution:*

\[
\begin{array}{c|c|c|c|c|c|c}
\text{Inputs} & 0.75 & 10 & -100 \\
\hline
\text{N} & \text{I} & \text{PV} & \text{PMT} & \text{FV} \\
\hline
\text{Output} & & & & = 107.41 \\
\end{array}
\]

Present values, annuities, and problems where you must find interest rates or numbers of periods with fractional time periods can all be handled with ease with a financial calculator or spreadsheet program. Indeed, financial calculators and spreadsheet programs have calendar functions specifically designed to deal with fractional time periods. However, any further discussion is beyond the scope of this text.

**DEBT (BOND) VALUATION**

We will use bonds to illustrate debt valuation, but the techniques discussed in the following sections are applicable to most types of debt.
The Basic Bond Valuation Model

Bonds call for the payment of a specific amount of interest for a specific number of years, and for the repayment of par on the bond’s maturity date. Thus, a bond represents an annuity plus a lump sum, and its value is found as the present value of the expected cash flow stream. To illustrate, Big Sky Healthcare has a 15-year bond, 10 percent bond outstanding, with a $1,000 par value. Here are the cash flows on a time line:

\[
\begin{array}{ccccccc}
0 & 1 & 2 & \ldots & 13 & 14 & 15 \\
\$100 & \$100 & \$100 & \$100 &-100 & \$100 & \$100 \\
1,000 & \\
\end{array}
\]

The value of the bond can be found using most financial calculators as follows:

Inputs: 15, 10, -100, -1000

<table>
<thead>
<tr>
<th>N</th>
<th>I</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>10</td>
<td>-100</td>
<td>-1000</td>
<td></td>
</tr>
</tbody>
</table>

Output = 1,000

Input N = 15, I = 10, PMT = -100, and FV = -1000, and then press the PV key to get the answer, 1,000. (The cash flows were treated as outflows so that the value would be displayed as a positive number.) Note that in bond valuation, all five time-value-of-money keys on a financial calculator are used because bonds involve both an annuity and a lump sum.

Zero Coupon Bonds

Some bonds, called zero coupon bonds, pay no interest at all during the life of the bond, so an investor’s cash flows consist solely of the return of par value at maturity. Because there are no interest payments, when the bond is issued its value is much less than par value, so the bond originally sells at a discount. Thus, zero coupon bonds also are called original issue discount bonds.

Zero coupon bonds are valued in the same way as regular (coupon) bonds, recognizing that there are no coupon payments to contribute to the bond’s value. To illustrate this concept, assume that Big Sky’s 15-year bond issue discussed in the previous section was a zero coupon bond. Assuming a 10 percent
required rate of return, the bond’s value would be $239.39:

\[
\text{Inputs} \quad 15 \quad 10 \quad -1000 \\
\text{N} \quad \text{I} \quad \text{PV} \quad \text{PMT} \quad \text{FV} \\
\text{Output} \quad = 239.39
\]

Note that this amount is merely the present value of the maturity payment that was calculated in the previous section.

Yield to Maturity

Up to this point, a bond’s required rate of return and cash flows have been used to determine its value. In reality, investors’ required rates of return on securities are not observable, but security prices can be easily determined—at least on those securities that are actively traded—by looking in the local newspaper or the Wall Street Journal. Suppose that the Big Sky bond had 14 years remaining to maturity, and the bond was selling at a price of $1,494.93. What percentage rate of return, or \textit{yield to maturity (YTM)}, would be earned if the bond was bought at this price, held to maturity, and no default occurred? To find the answer, 5 percent, use a financial calculator as follows:

\[
\text{Inputs} \quad 14 \quad 1,494.93 \quad -100 \quad -1000 \\
\text{N} \quad \text{I} \quad \text{PV} \quad \text{PMT} \quad \text{FV} \\
\text{Output} \quad = 5.00
\]

The YTM can be thought of as the expected rate of return on the bond. It is similar to the total rate of return discussed in the previous section. For a bond that sells at par, the YTM consists entirely of an interest yield, but if the bond sells at a discount or premium, the YTM consists of the current yield plus a positive or negative capital gains yield.

Yield to Call

Bonds that are callable have both a YTM and a \textit{yield to call (YTC)}. The YTC is similar to the YTM, except
that it assumes that the bond will be called. Thus, the YTC is calculated like the YTM, except that N reflects
the number of years until the bond will be called, as opposed to years to maturity, and M reflects the call
price, rather than the maturity value.

For example, suppose the Big Sky bond had ten years of call protection when it was issued. There
are now nine years to the date of first call, and the bond is selling at a price of $1,494.93. Furthermore, there
is a $100 call premium that must be paid if the issue were called at the earliest possible date. What YTC
would be earned if the bond were bought at this price and held to first call, at which time it was redeemed for
$1,000 + $100 = $1,100? The answer is 4.2 percent:

Inputs 4 1,494.93 -100 -1100
N  I  PV  PMT  FV
Output = 4.2

Bond Values with Semiannual Coupons

Virtually all bonds issued in the United States actually pay interest semiannually, or every six months. To
apply the preceding valuation concepts to semiannual bonds, the bond valuation procedures must be
modified as follows:

• Divide the annual interest payment, INT, by two to determine the dollar amount paid each six months.
• Multiply the number of years to maturity, N, by two to determine the number of semiannual interest
  periods.
• Divide the annual required rate of return, R(R), by two to determine the semiannual required rate of
  return.

To illustrate the use of the semiannual bond valuation model, assume that the Big Sky bonds pay
$50 every six months rather than $100 annually. Thus, each interest payment is only half as large, but there
are twice as many of them. When the going rate of interest is 5 percent annually, the value of Big Sky’s
bonds with 14 years left to maturity is $1,499.12:
Similarly, if the bond were actually selling for $1,400 with 14 years to maturity, its YTM would be 5.80 percent:

<table>
<thead>
<tr>
<th>Inputs</th>
<th>28</th>
<th>2.5</th>
<th>-50</th>
<th>-1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>PV</td>
<td>PMT</td>
<td>FV</td>
<td></td>
</tr>
</tbody>
</table>

Output = 1,499.12

The value for I, 2.90 percent, is the **periodic (semiannual) YTM**, so it is necessary to multiply it by two to get the annual YTM.